

6.4

Rhombuses, Rectangles, and Squares

Goal Use properties of special types of parallelograms.

VOCABULARY

Rhombus A rhombus is a parallelogram with four congruent sides.

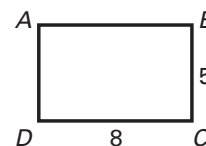
Rectangle A rectangle is a parallelogram with four right angles.

Square A square is a parallelogram with four congruent sides and four right angles.

Example 1 Use Properties of Special Parallelograms

In the diagram, $ABCD$ is a rectangle.

- Find AD and AB .
- Find $m\angle A$, $m\angle B$, $m\angle C$, and $m\angle D$.

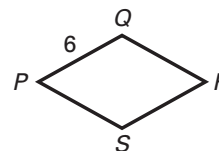
**Solution**

- By definition, a rectangle is a parallelogram, so $ABCD$ is a parallelogram. Because opposite sides of a parallelogram are congruent, $AD = \underline{BC} = \underline{5}$ and $AB = \underline{DC} = \underline{8}$.
- By definition, a rectangle has four right angles, so $m\angle A = m\angle B = m\angle C = m\angle D = \underline{90}^\circ$.

✓ **Checkpoint** Complete the following exercise.

- In the diagram, $PQRS$ is a rhombus. Find QR , RS , and SP .

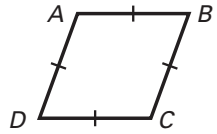
$$QR = 6, RS = 6, SP = 6$$



RHOMBUS COROLLARY

Words If a quadrilateral has four congruent sides, then it is a rhombus.

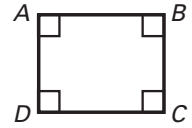
Symbols If $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$, then $ABCD$ is a rhombus.



RECTANGLE COROLLARY

Words If a quadrilateral has four right angles, then it is a rectangle.

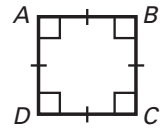
Symbols If $m\angle A = m\angle B = m\angle C = m\angle D = 90^\circ$, then $ABCD$ is a rectangle.



SQUARE COROLLARY

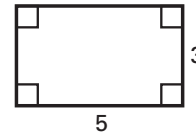
Words If a quadrilateral has four congruent sides and four right angles, then it is a square.

Symbols If $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $m\angle A = m\angle B = m\angle C = m\angle D = 90^\circ$, then $ABCD$ is a square.



Example 2 Identify Special Parallelograms

Use the information in the diagram to name the special quadrilateral.



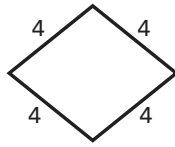
Solution

The quadrilateral has four right angles. So, by the Rectangle Corollary, the quadrilateral is a rectangle.

Because all of the sides are not the same length, you know that the quadrilateral is not a square.

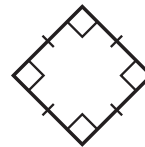
✓ **Checkpoint** Use the information in the diagram to name the special quadrilateral.

2.



rhombus

3.

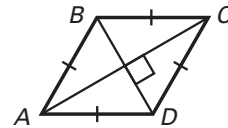


square

THEOREM 6.10

Words The diagonals of a rhombus are perpendicular.

Symbols In rhombus $ABCD$, $\overline{AC} \perp \overline{BD}$.



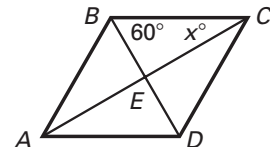
Example 3 Use Diagonals of a Rhombus

$ABCD$ is a rhombus. Find the value of x .

Solution

By Theorem 6.10, the diagonals of a rhombus are perpendicular. Therefore, $\angle BEC$ is a right angle, so $\triangle BEC$ is a right triangle.

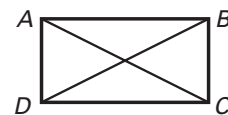
By the Corollary to the Triangle Sum Theorem, the acute angles of a right triangle are complementary. So, $x = 90 - 60 = 30$.



THEOREM 6.11

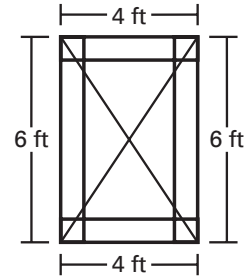
Words The diagonals of a rectangle are congruent.

Symbols In rectangle $ABCD$, $\overline{AC} \cong \overline{BD}$.



Example 4**Use Special Properties of Parallelograms**

- a. You nail four pieces of wood together to build a four-sided frame, as shown. What is the shape of the frame?
- b. The diagonals measure 7 ft 4 in. and 7 ft 2 in. Is the frame a rectangle?

**Solution**

- a. The frame is a parallelogram because both pairs of opposite sides are congruent.
- b. No. The frame is not a rectangle because the diagonals are not congruent.

Follow-Up

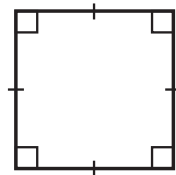
Which theorem did you use in Example 4, part (b)?

Theorem 6.11

- ✓ **Checkpoint** *ABCD* has the properties shown in the figure. Is the statement *true* or *false*? Explain.

4. *ABCD* is a rhombus.

True; it has four congruent sides.



5. *ABCD* is a parallelogram.

True; both pairs of opposite sides are congruent.

6. *ABCD* is a rectangle.

True; it has four right angles.

7. The diagonals are congruent.

True; it is a rectangle.